Justification of the technique used to calculate forward rates from zero rates by example.

This brief essay assumes that you have heard my Economics 136 lecture entitled *Interest Rate Theory* or have read chapter 4 of *Options, Futures, and Other Derivatives* by John C. Hull and have seen the mathematical derivation of ***zero rates*** and ***forward rates***.

The concept of ***zero rates*** is easy for students to understand. They treat each individual cash payment as a zero-coupon note or bond discounted back to the present using a continuous yield.

The concept of the ***forward rate*** is harder to understand. It represents the interest earned in *only that year*.

For example, the 3-year ***zero rate*** is the continuous rate earned over 3 years from a single payment earned at the end of 3 years. The 3-year ***forward rate*** is the interest earned in the third year alone.

Additionally, if interest rates conform to a positively-sloped yield curve, then the ***forward rate*** for that year will be above the ***zero rate*** for the same year. That seems counterintuitive to students.

What follows is a simple application **example** that may make the concept easier to grasp.

I lend money. Because I am in a competitive business I lend at market rates, and my customers and I agree to lend and borrow using continuous rates rather than discrete rates for our interest calculations.

Here is a table showing current rates:

Rate Table

Year	Zero	Forward
1	0.03	
2	0.04	0.05
3	0.05	0.07

You approach me because you want a special type of loan. You want to borrow $100 from me and pay me back exactly one year later. But you don't need the money today. You need the money in exactly two years and want to pay it back in a single payment in exactly three years. But you want the contract *now*. (This is a very unconventional loan but many types of forward contracts and swaps are structured in a similar way).

You ask me what interest I will charge at the end of the third year when the loan is paid back. I quickly look up the ***forward rate*** for year 3, which is 7%, and tell you that I will charge that rate and at the end of that year you will owe me $107.25. Because you understand how these rates work you agree.

My rate of interest earned *in the third year* will be 7%. (That is pretty obvious isn't it – I'm charging 7% continuously compounded and you have agreed to pay it). The interest that *I will have earned on my investment altogether over the three years* will equal 5%, the ***zero rate*** for 3 years.

Now let's investigate the logic of this contract.

First, remember that I have the option of simply investing my money in a loan now payable in three years that would earn the 3-year ***zero rate***. So I won't charge you a rate that will earn me less.

Second, because I have contracted to lend you $100 in two years, I need to set aside cash dedicated to this contract now. But I don't need to set aside $100. I need to set aside $92.31 and invest it for *two* years at the ***zero rate*** of 4%;

$$(1) \qquad 92.31e^{(0.04)\times 2} = 100$$

I then lend that to you at the 3-year ***forward rate*** of 7%, earning $7.52 interest;

$$(2) \qquad 100e^{(0.07)} = \$107.25$$

But before I do I had better check and make sure that I would make at least as much profit as I would have made if I had invested the $92.31 in a loan made today payable in three years at the 3-year ***zero rate***;

$$(3) \qquad 92.31e^{(0.05)\times 3} = \$107.25$$

My condition is satisfied.