

Yield-bearing Financial Assets (YBFAs)

The quiet market (until lately)

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Primary YBFA markets

- Note and Bond
 - U.S. Treasury notes and bonds
 - Corporate notes and bonds
 - Municipal (tax free) notes and bonds
 - Asset-backed securities (ABS)
- Money Market (maturities less than 1 year)
 - U.S. Treasury bills
 - Corporate commercial paper
 - Bank certificates of deposit

Some interesting facts about notes and bonds (hereafter bonds) ...

- Bonds are debt obligations of the party that issues them .. they are a debt contract that promises to pay period interest payments over so many years then the full principal at the end.
- Bonds are bought and sold on a huge secondary market.
- Bonds are first issued by the agency that issues them at or very near par (100), but thereafter trade above or below par.
- **Bond values on the secondary market move in opposite directions of interest rates! When interest rates rise, bond values fall! This is called market risk.**

U.S. Treasury Securities

U.S Treasury Securities Offered to the Public		
Security	Maturity	Now Offered
Bills:	Less than one year	4,13,26 and 52 weeks
Notes:	More than one to ten years	2,3,5,7,9 and 10*
Bonds:	20 to 30 years	30 years*
Inflation Indexed:	5, 10, and 20 years	All*

*These are sometimes approximate. E.g. a 30 year bond might have a maturity of 29 years and 11 months.

The budget deficit is financed by the sale of interest-bearing U.S. Treasury securities to the public, including corporations, financial institutions, and foreign investors. The securities differ largely by the maturities. After a 5-year hiatus, the Treasury began selling 30 year bonds in February 2006. The Treasury in recent years has sold 20 year bonds also, but is not currently selling them. Inflation-indexed securities are called TIPS, and are described in the reading material for this section.

Total U.S. Securities Outstanding & Gross and Net Debt; June 2011

Instrument	Amount
Marketable	9,314.9
Bills	1,529.4
Notes	6,146.0
Bonds	974.2
Inflation-indexed	665.3
Non-Marketable	5,052.7
Government Account	4,596.2
Non-marketable privately held	427.4
TOTAL PUBLIC DEBT:	14,367.6
NET PUBLIC DEBT:	9,771.4
(Net is total less govt. account)	

Source: U.S. Treasury Bulletin, September 2011, Tables FD-1 and FD-2.

Who owns the U.S. Treasury Debt?

Ownership of Privately Held U.S. Treasury Debt

	December 1995		March 2011	
	Billions \$	% total	Billions \$	% total
Deposit institutions	315.4	9.5%	328.7	4.0%
US Savings Bonds	155.0	4.7%	186.7	2.2%
Private pension funds	142.9	4.3%	637.7	7.7%
S&L gov't pension funds	208.2	6.3%	187.9	2.3%
Insurance companies	241.5	7.3%	248.8	3.0%
Mutual funds	225.1	6.8%	643.4	7.7%
State & local governments	289.8	8.8%	506.1	6.1%
Foreign holdings	835.2	25.3%	4,479.2	53.9%
Other (mostly individuals)	864.6	26.1%	1,092.5	13.1%
Total privately held	3,307.7	100.0%	8,311.1	100.0%
Memo:				
Total Debt:			14,270.0	
Held by FRS & IG accounts:			5,958.9	41.8%

Source: U.S. Treasury Bulletin, September 2011, Table OFS-2

Original Treasury Issue: Dutch Auctions

When U.S. Treasury securities first come to the market, they are sold at scheduled auctions throughout the year. The frequency of the auction varies by the type of security, ranging from once a week (every Thursday) for 13- and 26-week bills to twice a year for 30-year bonds. See

<http://www.treasurydirect.gov>

under institutions for the calendar of Treasury auctions.

Prior to the auction, the Treasury will set the size of the subscription for the type of security and announce it publicly (e.g. \$10 billion of 2-year notes will be sold on March 15). A private investor can buy some of the subscription by submitting a **non-competitive tender** for an amount up to \$5 million through **Treasury Direct** (<http://www.treasurydirect.gov>).

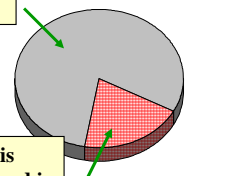
Larger investors are required to submit a **competitive bids tender** to participate in a **dutch auction**. The competitive bidder will submit based upon the *lowest yield that she is willing to accept up to three decimal places*.

How the Dutch Auction Works

In this example, the Treasury is selling \$10 billion worth of 2-year notes. Tenders are received for \$2 billion of non-competitive bids (gray area).

Tenders are also shown for the competitive bids (green and yellow) totaling \$15 billion. Each bidder bids the *minimum* yield that he will accept up to 3 decimal places. Starting with the lowest bid the auction works up until the competitive subscription is filled at \$8 billion. Winners (green) are awarded the *highest* yield accepted. Those who bid at the cutoff are pro-rated.

80% is competitive bid



The other \$2 million is non-competitive bid and is awarded at high bid.

Amount	Bids	Result
2	4.389	Rejected
3	4.388	Rejected
4	4.387	Pro-rated 50% at 4.387
2	4.386	Accepted at 4.387
1	4.385	Accepted at 4.387
2	4.384	Accepted at 4.387
1	4.383	Accepted at 4.387
2	NC	Accepted at 4.387

Tentative Auction Schedule of U.S. Treasury Securities				
Security Type		Announcement Date	Auction Date	Settlement Date
3-Year NOTE		Wednesday, August 04, 2010	Tuesday, August 10, 2010	Monday, August 16, 2010
10-Year NOTE		Wednesday, August 04, 2010	Wednesday, August 11, 2010	Monday, August 16, 2010
30-Year BOND		Wednesday, August 04, 2010	Thursday, August 12, 2010	Monday, August 16, 2010
13-Week BILL		Thursday, August 05, 2010	Monday, August 09, 2010	Thursday, August 12, 2010
26-Week BILL		Thursday, August 05, 2010	Monday, August 09, 2010	Thursday, August 12, 2010
4-Week BILL		Monday, August 09, 2010	Tuesday, August 10, 2010	Thursday, August 12, 2010
13-Week BILL		Thursday, August 12, 2010	Monday, August 16, 2010	Thursday, August 19, 2010
26-Week BILL		Thursday, August 12, 2010	Monday, August 16, 2010	Thursday, August 19, 2010
4-Week BILL		Monday, August 16, 2010	Tuesday, August 17, 2010	Thursday, August 19, 2010
13-Week BILL		Thursday, August 19, 2010	Monday, August 23, 2010	Thursday, August 26, 2010
26-Week BILL		Thursday, August 19, 2010	Monday, August 23, 2010	Thursday, August 26, 2010
52-Week BILL		Thursday, August 19, 2010	Tuesday, August 24, 2010	Thursday, August 26, 2010
30-Year TIPS	R T	Thursday, August 19, 2010	Monday, August 23, 2010	Tuesday, August 31, 2010
2-Year NOTE		Thursday, August 19, 2010	Tuesday, August 24, 2010	Tuesday, August 31, 2010
5-Year NOTE		Thursday, August 19, 2010	Wednesday, August 25, 2010	Tuesday, August 31, 2010
7-Year NOTE		Thursday, August 19, 2010	Thursday, August 26, 2010	Tuesday, August 31, 2010
4-Week BILL		Monday, August 23, 2010	Tuesday, August 24, 2010	Thursday, August 26, 2010
13-Week BILL		Thursday, August 26, 2010	Monday, August 30, 2010	Thursday, September 02, 2010
26-Week BILL		Thursday, August 26, 2010	Monday, August 30, 2010	Thursday, September 02, 2010
4-Week BILL		Monday, August 30, 2010	Tuesday, August 31, 2010	Thursday, September 02, 2010
10-Year TIPS	R T	Monday, August 30, 2010	Thursday, September 02, 2010	Wednesday, September 15, 2010
13-Week BILL		Thursday, September 02, 2010	Tuesday, September 07, 2010	Thursday, September 09, 2010
26-Week BILL		Thursday, September 02, 2010	Tuesday, September 07, 2010	Thursday, September 09, 2010
3-Year NOTE		Thursday, September 02, 2010	Tuesday, September 07, 2010	Wednesday, September 15, 2010
10-Year NOTE	R	Thursday, September 02, 2010	Wednesday, September 08, 2010	Wednesday, September 15, 2010
30-Year BOND	R	Thursday, September 02, 2010	Thursday, September 09, 2010	Wednesday, September 15, 2010

Treasury Bond/Note Quotations

Sample quotation from *finance.yahoo.com* taken October 2010

Type	Issue	Price	Coupon(%)	Maturity	YTM(%)	Current Yield(%)
Treas	T-NOTE 4.375 15-Aug-2012	98.25	4.375	15-Aug-12	4.693	4.453
Treas	T-NOTE 4.000 15-Nov-2012	95.99	4.000	15-Nov-12	4.706	4.167
Treas	T-NOTE 3.875 15-Feb-2013	95.11	3.875	15-Feb-13	4.709	4.074

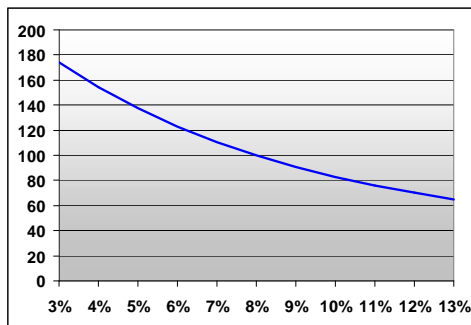
These notes have a **par** value of 100.

1. The **coupon rate** is the annual interest payment *based upon par*.
2. The **price** is the *ask* that you will pay (per \$100) if you buy this bond.
3. The **yield to maturity** is the yield that you will earn if you buy at this **price** and are paid this **coupon rate**. It includes the yield that is implicit in the price appreciation of this bond (i.e. if you buy it for 95.11 and it matures at 100).
4. The **current yield** is the **price** divided by the **coupon rate**, which does not take into account any price appreciation or depreciation.

These notes are trading at **discount** because their price is below par. When they trade above par they are trading at a **premium**.

Treas	T-NOTE 3.875 15-Feb-2013	106.00	3.875	15-Feb-2013	0.535	3.655
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October 13, 2011



As this slide will show later in the lecture, even U.S. Treasury Bonds have market risk.

Bond Values and Interest Rates

... a contrary relationship

YBFA yields

- **Two types of payouts**
 - Coupon payment (bonds and notes)
 - Discount (bills, where yield is implicit)
- **Three yield calculations that you must know**
 - Coupon yield (not calculated, stated)
 - 1. **Discount yield** and discount price (bills only)
 - 2. **Current yield**
 - 3. **Yield to maturity** (also called **Ask Yield**)

Coupon Yield

U.S. Treasury Notes and Bonds

A note or bond's coupon rate is the rate declared when the bond is issued, expressed as a percentage of par. U.S. Treasury Notes and Bonds pay interest twice per year, according to the following formula:

$$\text{Coupon payment} = (\text{par value} \times \text{coupon rate}) / 2$$

The interest payment for the 3.875 Feb 2013 note would be

$$(100 \times 0.03875) / 2 = \$1.9375 \text{ semi-annually per 100}$$

Type	Issue	Price	Coupon(%)	Maturity	YTM(%)	Current Yield(%)
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Treas	T-NOTE 3.875 15-Feb-2013	95.11	3.875	15-Feb-13	4.709	4.074

1. Discount yield

for bills (money market financial assets)

Discounting MMFAs: Buy at a discounted price (below par, 100), sell at par, interest is implicit in the appreciation.

$$\text{Yield} = \frac{100 - \text{price}}{\text{price}} \times \frac{365}{\text{days to maturity}}$$

For example, a 26-week bill selling at 96.16 yields 8%:

$$8\% = \frac{100 - 96.16}{96.16} \times \frac{365}{182}$$

Discount price (when yield is known) for money market financial assets (continued)

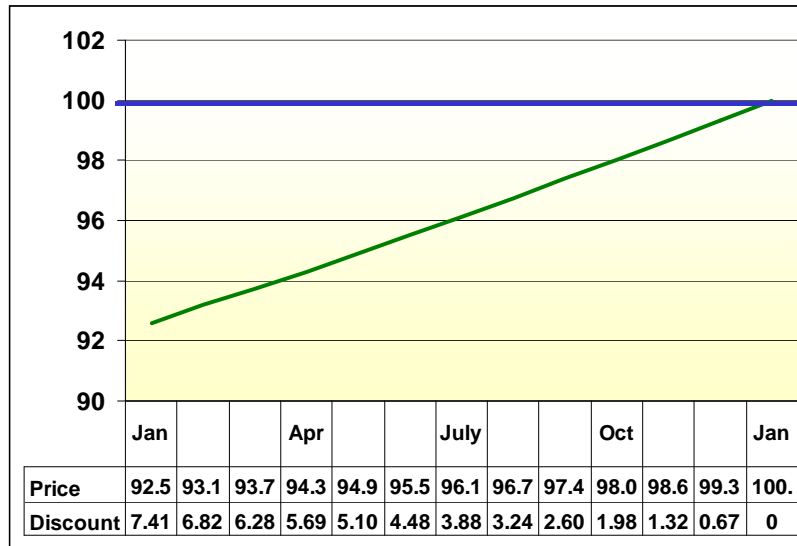
The formula for determining price when the yield is known:

$$P = \frac{Par}{\left[\left(yield \times \left(\frac{dtm}{365} \right) \right) \right] + 1}$$

Same example:

$$Price = \frac{100}{\left(yield \times \left(\frac{dtm}{365} \right) \right) + 1} = \frac{100}{\left(.08 \times \left(\frac{182}{365} \right) \right) + 1} = 96.16$$

Projected Prices – 52 week bill @ 8%



2. Current yield

Because you will not be paying par value for the bond or note that you buy, your short-term yield is current yield:

$$\text{Current yield} = \frac{\text{coupon rate} \times \text{par}}{\text{ask price}}$$

Using our example of the Feb note with a coupon of 3.875 priced at 110.30:

$$4.074\% = 3.875\% / 95.11$$

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A question??

- Ten years ago you bought a new bond with the following features:
 - **\$1000** even (par 100)
 - 30 year bond (20 years remaining)
 - Coupon rate was **5%** (\$50 yearly)
- Today 20 year bonds are yielding **10%**
- Question: Can you resell your bond today ..
 - at par (**\$1,000**)?
 - at any price?

Price/Yield Tradeoff

... an example

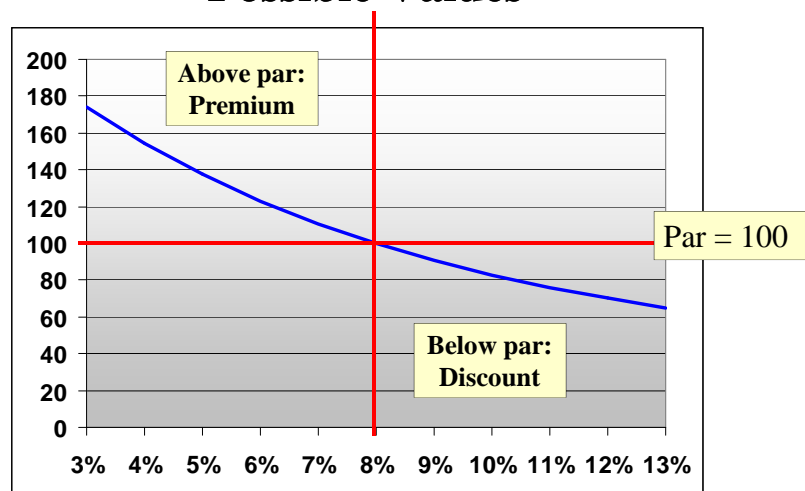
Suppose you buy a newly-issued 30-year bond which has a coupon of **8%**.

Ten years pass. Your 30-bond is now the equivalent of a 20-year bond (it has the same cashflow characteristics). Therefore if you wish to sell it on the secondary market, it must offer an **ask yield**, also called **yield to maturity**, that is competitive with a newly-issued 20-year bond.

What if the 20-year bond coupon is **6%**? Or **11%**? What will your old 30-year bond be worth?

The range of possibilities is shown on the next slide.

Possible Values



This shows the full range of ask-yields (ytm) possible for an original 30-year bond with 8% coupon 10 years later when compared to a 20-year bond with possible yields shown on the bottom axis.

What is a bond or note worth??

A bond or note is nothing more than a future stream of cash. **A bond or note is worth the present discounted value of all future cash payments.** That's it! And this is not an approximation, this is a mathematical certainty that defines the relationship between a bond's market price and its effective yield.

Sometimes the value of future cash payments is known (Treasuries) and sometimes estimated (bonds with default possibilities).

Therefore, to understand the pricing of bonds and notes, you must understand the math of compounding and present value calculations.

The Compounding Formula

What is the formula for calculating the future value X_f of the present value X_p invested at interest rate r (compounded annually) for n years?

$$X_f = (1 + r)^n \times X_p$$

\$10 invested for 5 years at an 8% compounded rate will be worth:

$$10(1.08)^5 = \$14.69$$

The Present Value Formula

What is the present value X_p of some guaranteed future value X_f , assuming the opportunity to invest money today at some compound interest rate r ?

$$X_p = \frac{X_f}{(1+r)^n}$$

For example, what is the present value of a promise to pay \$10 all at once at a date 5 years in the future, if money today can be expected to earn 8% between now and then?

$$\$6.81 = \frac{10}{(1.08)^5}$$

The present value of a payments stream

What is the present value of a promise to pay two payments of \$100 each at a date 5 years in the future and again 10 years in the future if money today can be expected to earn 12% between now and then?

$$PV = \sum_{i=1}^2 \frac{100}{(1+.12)^{5i}} = \frac{100}{1.12^5} + \frac{100}{1.12^{10}} = 88.94$$

What is a bond or note worth?

To be precise, a bond or note is worth the value of every cash payment that it will make, individually discounted to the present and summed.

For example, a newly issued U.S. Treasury \$10,000 ten year note with a coupon of eight percent (8%) will be paying to the holder twenty payments of \$400 each every six months and a single payment of \$10,000 at the end of ten years.

This note will therefore be worth the present discounted value of these 21 payments, summed.

Valuing a bond (simple example)

To value a bond (or note) on the secondary market, you will need to know the coupon rate (cr), the time remaining in the life of the bond (t), and the present day yield (y) of equivalent bonds.

What would be today's value of a 30-year bond issued 10 years ago at a coupon rate of 12% (cr) if other bonds with 20 years (t) remaining until maturity today yield only 6% (y)? (Assume annual rather than semi-annual interest payments).

$$PV = \sum_{i=1}^{20} \frac{cr \times 100}{(1+y)^i} + \frac{100}{(1+y)^{20}} = \sum_{i=1}^{20} \frac{12}{1.06^i} + \frac{100}{1.06^{20}}$$

The simple bond valuation formula

$$MV = \sum_{i=1}^n \frac{C}{(1+r)^i} + \frac{Par}{(1+r)^n}$$

where

MV = market value *presently* of the bond

n = number of years to maturity

C = coupon payment (**par times the coupon interest rate**)

r = *present* yield of this bond (market determined)

This formula assumes that there is only one interest payment per year and that this bond was priced on the day after the most recent interest payment was made.

... another way of writing it

$$MV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots + \frac{C}{(1+r)^n} + \frac{Par}{(1+r)^n}$$

The present discounted value of each of the coupon interest payments.

The present discounted value of the redemption value of the bond at par.

A yield-bearing financial asset is worth the present discounted value of its future cashflow, which consist of interest payments and redemption value.

The elementary bond formula (reduced form)

$$MV = (CR \times 100) \times \left(\frac{1 - \left[\frac{1}{(1+r)^n} \right]}{r} \right) + \left(100 \times \frac{1}{(1+r)^n} \right)$$

where

MV: the present market value of the bond
CR: coupon rate (original yield) of the bond
r: current market rate on equivalent bonds
n: number of remaining years to maturity

The original formula is a geometric series and this is a reduced-form equation. To see its justification and derivation, read the appendix in the reading assigned for this lecture.

Note: This formula cannot be used to value an actual bond because it assumes only one interest payment per year and that the bond is being bought on the day of the coupon payment. For actual bond pricing a more complicated version of this is shown at the end of this lecture.

... a bond at premium

$$137.39 = (.08 \times 100) \times \left(\frac{1 - \left[\frac{1}{(1+.05)^{20}} \right]}{.05} \right) + \left(100 \times \frac{1}{(1+.05)^{20}} \right)$$

where

MV: 137.39
CR: 8%
r: 5%
n: 20

This shows the current market value (137.39) of a 30-year bond that was issued 10 years ago (and hence has 20 years of life left) and pays a coupon interest rate of 8% per year (determined at its time of issue) given that interest rates now on an equivalent bond (a 20-year bond) are only 5%.

... a bond at discount

$$82.97 = (.08 \times 100) \times \left(\frac{1 - \left[\frac{1}{(1+.10)^{20}} \right]}{.10} \right) + \left(100 \times \frac{1}{(1+.10)^{20}} \right)$$

where
MV: 82.97
CR: 8%
r: 10%
n: 20

This shows the current market value (82.97) of a 30-year bond that was issued 10 years ago (and hence has 20 years of life left) and pays a coupon interest rate of 8% per year (determined at its time of issue) given that interest rates now on an equivalent bond (a 20-year bond) are now 10%.

You **must** understand the message of these next two slides.

Maturity and Volatility (Risk)

Security	Par	Original Yield	One Year Later	Value	% loss of Value
5-year note:	100	5%	6%	96.53	3.47
10-year bond:	100	5%	6%	93.20	6.80
30-year bond:	100	5%	6%	86.41	13.59

Using our formula, this shows that the longer the maturity of the bond, given any interest rate increase, the greater the percentage of the capital loss.

We assume that all three of these notes and bonds were originally issued at 5%. We assume one year later that yields on equivalent securities increased to 6%. Look at the percentage value of the capital loss.

This implies that the longer the maturity of a note or bond, the greater its sensitivity to interest rate changes, hence *the higher its perceived risk*.

... a more realistic example (typical yield curve)

Security	Par	Original Yield	One Year Later	Value	% loss of Value
5-year note:	100	5%	6%	96.53	3.47
10-year bond:	100	6%	7%	93.48	6.52
30-year bond:	100	7%	8%	88.48	11.52

This shows the more realistic example of these three securities being issued at coupon rates consistent with a typical yield curve. It is then assumed that the entire yield curve shifts up by one percent.

You get the same general result: the longer the maturity the greater the sensitivity to interest rates changes.

The formula when interest is paid semi-annually

$$MV = \sum_{i=1}^m \frac{C}{(1 + r/2)^i} + \frac{Par}{(1 + r/2)^m}$$

where

MV = market value

C = coupon interest payment (**par X r/2**)

r = present market yield

m = number of coupon payments remaining (years X 2)

3. The final formula ... ask yield (ytm)

$$MV = \frac{C}{m} \sum_{i=1}^n \frac{1}{\left(1 + \frac{r}{m}\right)^{i-a/p}} + \frac{Par}{(1+r)^{n/m-a/p}} - \left(\frac{C}{m} \times \frac{a}{p}\right)$$

This term represents an adjustment for accrued interest.

MV = market value, the quoted *ask* price of the bond,
 C = the annual coupon payment, equal to the coupon rate times par,
 $Par = 100$
 r = the prevailing annual market yield, expressed as *ask yield* or *yield to maturity*,
 m = the number of coupon payments per year,
 n = the number of remaining coupon payments,
 p = the number of days in *this* coupon period (between 181-184, use 182 if unknown),
 a = the number of days between the last coupon payment and the settlement day.

What is this term?

$$\left(C \times d/182\right)$$

When you buy a bond, you will owe accrued interest to the seller. It will be the interest accrued since the last coupon payment and it will be equal to this amount above. This value is added to the bonds quoted value so that bond prices will not exhibit a saw-tooth pattern, rising as the coupon payment date approaches then plunging the day after.



an example

You are buying a 30-yr bond that was issued on September 15, 1999:

Purchase date: February 8, 2009 (146 days since last coupon payment, 36 to next),

Next coupon date: March 15, 2009 (the first of 40 coupons remaining),

Redemption date: September 15, 2029 (for par and last coupon),

Coupon rate/amount: 8% yielding \$4 per coupon payment,

Present market rate (*ask* yield): 10%.

$$\frac{a}{p} = \frac{146}{182} = 0.8022$$

$$MV = \frac{8}{2} \sum_{i=1}^{40} \frac{1}{\left(1 + \frac{.10}{2}\right)^{i-0.8022}} + \frac{100}{(1+.10)^{20-0.822}} - \left(\frac{8}{2} \times 0.8022\right)$$

$$MV = 4 \sum_{i=1}^{40} \frac{1}{(1.05)^{i-0.8022}} + \frac{100}{(1.10)^{19.1978}} - (3.21) = 84.213$$

Reduced-form Version of the Complex Formula

$$MV = \frac{C}{m} \left[\frac{1 - \frac{1}{\left(1 + \frac{r}{m}\right)^n}}{\frac{r/m}{\left(1 + \frac{r}{m}\right)^{a/p}}} \right] + \left(Par \times \frac{1}{(1+r)^{n/a/p}} \right) - \left(\frac{C}{m} \times \frac{a}{p} \right)$$

Solving the same problem from the last slide (rounding error explains the difference):

$$MV = 4 \left[\frac{1 - \frac{1}{(1.05)^{40}}}{\frac{0.05}{(1.05)^{0.8022}}} \right] + \left(100 \times \frac{1}{(1.10)^{20-0.8022}} \right) - (4 \times 0.8022) = 84.23$$

Using the Bond Formula Calculator in your Homework

Treasury Bond Formula	
Version 1.1 May 24, 2010	
Today:	10/19/2010
Bond or CUSIP#:	15 May 2039
Maturity Date:	5/15/2039
Start Date:	5/24/2010
Coupon Rate:	0.04250
Current Ask Yield:	0.04125
Remaining Time in Years:	28.9945
Remaining # of Coupons:	58
Semi-annual Coupon:	2.125
PV Coupons:	71.500
PV Redemption:	30.603
Bond Value:	102.103
less Accrued Interest:	0.023
Final Value:	102.080

Equivalent to Formula 15 in bond handout.
 You may have to hit <F9> to recalculate values.