Fermat's Enigma
by Simon Singh

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I originally ran across the book *Fermat's Enigma* by Simon Singh [1] one day while I was at work (I work in the music department at Barnes & Noble), and was immediately interested. I had never heard of Fermat's Last Theorem, and was unfamiliar with the concept of mathematical proofs outside of proving triangles congruent in my high school geometry class. It was amazing to me that any sort of mathematical statement could create such an amazing series of events over the course of its 350 year history. Like countless others before me, the concept seemed simple enough: prove Fermat's Lat Theorem, which states that $x^n + y^n = z^n$ has no non-trivial whole number solutions for $n > 2$. However, as the book describes, this particular proof turned out to be the "mathematical equivalent of splitting the atom." To do the book justice, I will not attempt to paraphrase all of the stories and concepts it presents; I will merely highlight some of the more interesting aspects of an amazing mathematical journey through the life of a theorem.

Educationally, there are many concepts discussed in this book which apply to various mathematical pursuits. There are many fine descriptions and examples of proofs by induction, counterexample, and contradiction, and before taking Discrete Math, I had never read a more accurate explanation of the difference between scientific proofs and mathematical proofs as the one found in the very first chapter.

Historically, this book covers not only Fermat's life, Wiles' life, and the life and proof of the Theorem, but it also provides brief but thorough explanations of the lives of various mathematicians whose contributions made the final proof possible. One of the most interesting stories is that of Paul Wolfskehl, a rich German industrialist and amateur number theorist whose plans to kill himself were aborted when he entered a library on the night he was to kill himself at midnight (because of a woman, not mathematics). While he was wandering around the library, Wolfskehl came across a book containing the last attempt at a proof (made by Ernst Kummer) and found a flaw in it. Thinking he would finally be the one to complete the proof, Wolfskehl worked all through the night trying to correct Kummer's mistake. Although he did not complete the proof, Wolfskehl had passed the time he had planned for his death, and decided that it was the theorem that had saved his life. Upon his death in 1908 (from natural causes), Wolfskehl left a large portion of his estate (100,000 Marks, the equivalent of a million dollars today) to the person who proved the theorem true (although he left nothing to anyone that might prove it false).

It was through a conjecture made by two Japanese mathematicians, Goro Shimura and Yutaka Taniyama, stating that modular forms and elliptic equations were one and the same (a concept I'm certainly not clear on) and the work of American Ken Ribet linking the conjecture with Fermat's Theorem that a major breakthrough was made in the proof. However, the proof that the Taniyama-Shimura conjecture and Fermat's Last Theorem were the same did not provide a proof that they were true. In fact, Taniyama committed suicide because he was unable to prove the conjecture. It was based on this connection, however, that Andrew Wiles, a professor at Princeton University who had been obsessed with proving Fermat's Theorem since he was 10 years old, was able to make history by providing a proof on June 23rd, 1993.

Beginning in 1986, Wiles worked on the proof by himself, a break from standard practice in mathematics, where outcomes are based upon the ability of mathematicians to work together and build off of each other's ideas (another very interesting concept to me). For the next six years, he worked on his own without alerting anyone other than his wife to the fact that he was working on a proof. It wasn't until 1992, when he began using a method he didn't have complete control over, that Wiles enlisted the help of another professor at Princeton, Nick Katz, to aid him in the final steps of the proof. In order to keep the proof secret while still allowing himself the time and facilities to explain his work to Katz, Wiles created a lecture series that he offered as a class for graduate students, in which Katz enrolled. After the first few class periods, all students other than Katz had dropped the class, leaving the two to work on their own. Through

Katz's assistance and using countless techniques developed throughout the history of mathematics, Wiles was able to finish the proof and present it to the world at a conference in Cambridge (Wiles' home town) at the Isaac Newton Institute.

One of the greatest quotes I have ever found was at the beginning of the chapter describing the aftermath of the proof presentation: "A problem worthy of attack proves its worth by fighting back," by Piet Hein. In order to verify the proof, a committee of six people was established (compared to the normal review board of three); it consisted of the top mathematicians in the world, including Nick Katz, who had helped Wiles with the proof. Upon examination of Wiles' proof, Katz found an error that could not be corrected involving an assumption that was not verified. This was a major shock to Wiles and Katz, especially since a Japanese mathematician named Yoichi Mayaoka had announced in 1988 that he had proven the theorem himself, only to be proven wrong due to an equally small assumption. Knowing that if the proof were published in its current state, the error would be corrected by someone who would then receive all of the credit (another fascinating aspect to this story), Katz kept the knowledge of the error between himself, Barry Mazur (the head of the committee), and Wiles, who went to work correcting the error. This proved to be increasingly difficult, especially as rumors got around that the proof was now invalid. Feeling even more pressure than before, Wiles refused to give up, and spent the next 14 months agonizing over the correction.

In early April of 1994, while Wiles was working continuously to correct his error, it was announced over email that a counterexample had been found by Harvard professor Noam Elkies. Elkies had previously found a counterexample to a conjecture made by Euler. This announcement shocked the mathematics community, especially because of the almost rabid attention given to the error in Wiles' proof. However, upon examination of the emails that these professors were receiving, it was realized that while many of them were received on April 2nd or 3rd, Elkies' email originated on April 1st, April Fool's Day. The realization that the entire email was a hoax helped to break the tension surrounding the proof and also helped to teach a lesson to the more aggressive of the rumor perpetrators. It was in October of 1994 that Wiles was able to correct the error and complete his proof. Combined with the attention he received after his initial presentation of the proof, Wiles became a celebrity (even being named one of *People* magazine's "25 Most Intriguing People").

In addition to teaching me about numerous aspects of number theory and discrete mathematics, this book showed me in profound ways how much mathematics is built around the ability to use the work of previous generations to break new ground. In a way, this feature of mathematics requires of its player a strong understanding of history and ability to research endeavors of previous or current mathematicians (no matter how obscure) just as much as an ability to perform complex calculations or have phenomenal insights. It is through this interdisciplinary strength that a journey such as that of Fermat's Last Theorem can reach a conclusion.

Reference:

[1] Singh, Simon. *Fermat's Enigma*. New York: Anchor Books, 1998.